Framework for Deep and Temporal Complex-Valued Networks

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Abstract

We present a framework for building deep and temporal complex-valued networks that contain compositions of holomorphic and non-holomorphic functions. The gradients of a real-valued cost function are back-propagated through the network to adjust the parameters by:

- Using convenience of Wirtinger calculus.
- Leveraging identities of the complex conjugate.

Motivation

Back-propagation for complex valued networks. Easily build deep networks having compositions of holomorphic and non holomorphic functions.

Why fully-complex activations? We may want to have a means of jointly modeling phase-amplitude dependencies for time-frequency representations of audio.

Why split-complex activations? Liouiville's theorem: Every bounded entire function must be constant. We might prioritize boundedness in some components of a model.

Wirtinger Calculus

Let $z \in \mathbb{Z}$, $x, y \in \mathbb{R}$, with

$$f(z) = f(z, \bar{z}) = f(x, y) = u(x, y) + iv(x, y)$$

and $\bar{z}$ denoting complex conjugate of z.

$\mathbb{CR}$ **(Wirtinger) Calculus**:

$\mathbb{R}$-derivative $= \left.\frac{\partial f}{\partial z}\right|_{\bar{z} \text{ constant}}$ $\overline{\mathbb{R}}$-derivative $= \left.\frac{\partial f}{\partial \bar{z}}\right|_{z \text{ constant}}$

Cauchy-Riemann Equations:

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$, or equivalently: $\frac{\partial f}{\partial \bar{z}} = 0$.

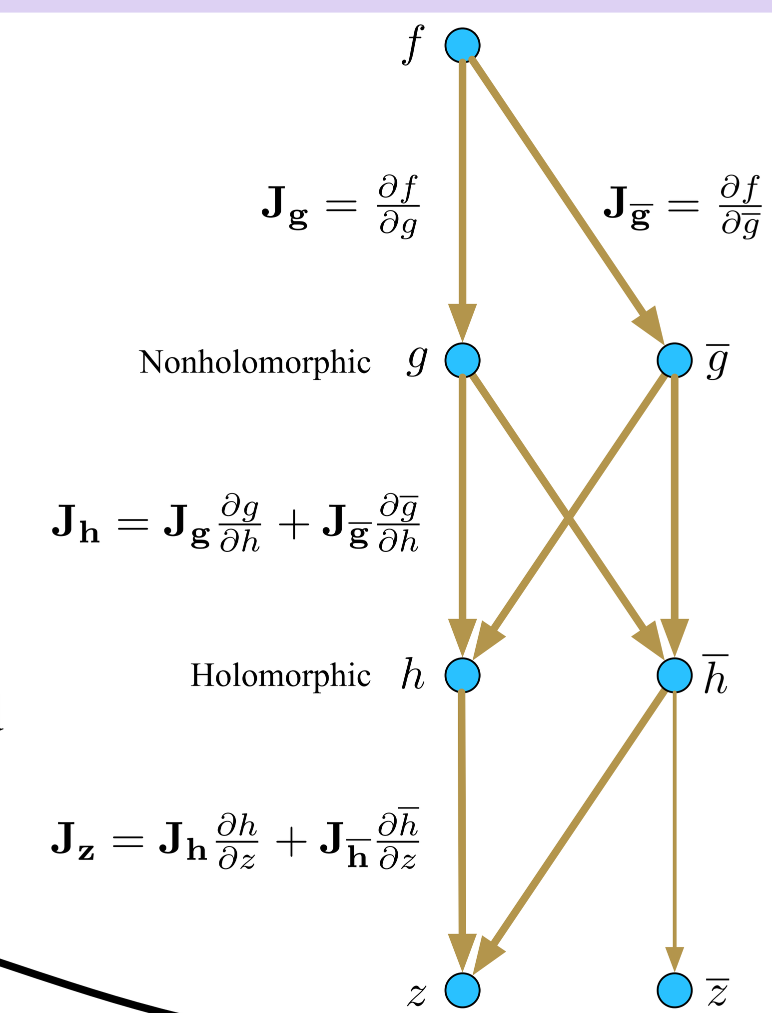
Framework

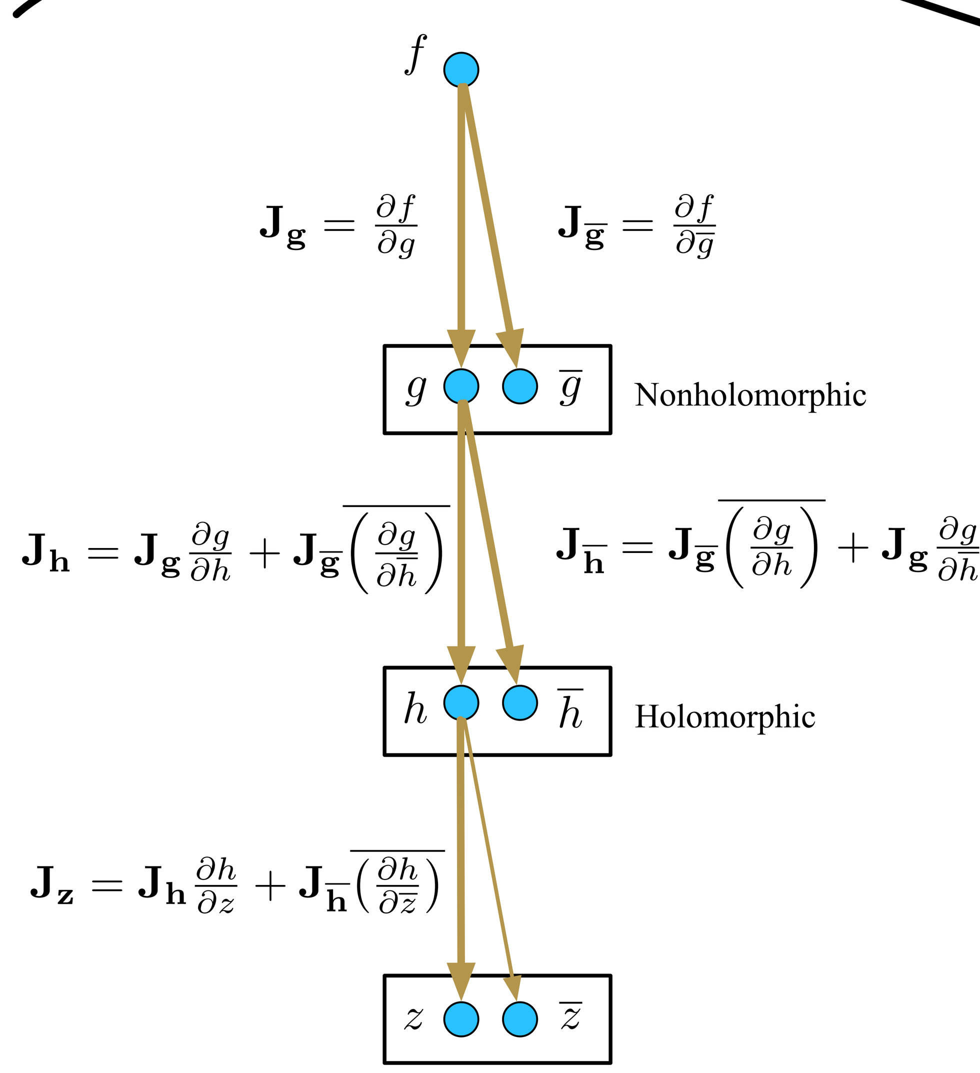
Dependency Graph:

$$y = f(g) = f(g, \bar{g}) = |g|^2 = g\bar{g}$$
$$y = (f \circ g \circ h)(z)$$

We seek to find the Jacobian of f with respect to z, $\mathbf{J_z}$.

If g or h are nonholomorphic then the dependency graph is not straightforward to compute.





Algorithm
Wirtinger calculus provides convenient means to compute partial derivatives of complex-valued functions.

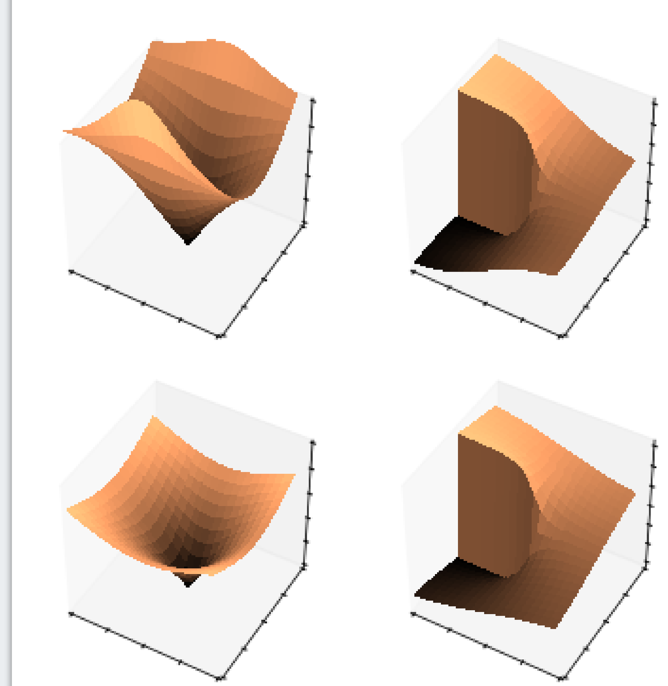
Use the following identities to keep track of two gradients rather than four:

$\frac{\partial \bar{f}}{\partial \bar{z}} = \overline{\left(\frac{\partial f}{\partial z}\right)}$, $\frac{\partial \bar{f}}{\partial z} = \overline{\left(\frac{\partial f}{\partial \bar{z}}\right)}$.

$\frac{\partial f}{\partial \bar{z}} = 0$ when f is holomorphic.

Activation Functions

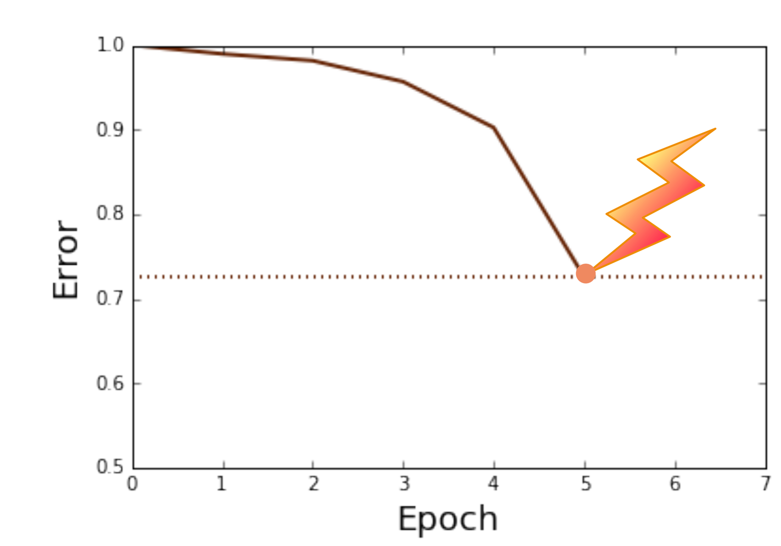
Elementary transcendental functions have desirable characteristics [2] but may be difficult to train.



tanh: holomorphic but unbounded

Squashing and phase-preserving [3]: not holomorphic but bounded

Complex-valued LSTM encounters singularity during training on toy task.



Acknowledgments

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